

4.

Problem 11.46P (HRW)

The oxygen molecule, O_2 , has a total mass of 5.30×10^{-26} kg and a rotational inertia of 1.94×10^{-46} kg m² about an axis through the centre of the line joining the atoms and perpendicular to that line. Suppose that such a molecule in a gas has a speed of 500 m s^{-1} and that its rotational kinetic energy is two-thirds of its translational kinetic energy. We have to find its angular velocity.

Solution:

Mass of the oxygen molecule, $M_{O_2} = 5.30 \times 10^{-26}$ kg.

Its rotational inertia, $I = 1.94 \times 10^{-46}$ kg m².

Speed of the molecule, $v = 500 \text{ m s}^{-1}$.

Its kinetic energy of translation,

$$KE_{tr} = \frac{1}{2} M_{O_2} v^2 = \frac{1}{2} \times 5.30 \times 10^{-26} \times 500^2 \text{ J} = 6.625 \times 10^{-21} \text{ J}.$$

In the problem it has been given that the rotational kinetic energy of the molecule is two-thirds of its translational kinetic energy. That is

$$KE_{rot} = \frac{2}{3} \times 6.625 \times 10^{-21} \text{ J} = 4.416 \times 10^{-21} \text{ J}.$$

Rotational velocity, ω , of the molecule can be calculated from the relation

$$KE_{ro} = \frac{1}{2} I \omega^2.$$

Therefore,

$$\begin{aligned} \omega &= \sqrt{2KE_{ro}/I} = \sqrt{2 \times 4.416 \times 10^{-21} / 1.94 \times 10^{-46}} \\ &= 6.7 \times 10^{12} \text{ rad s}^{-1}. \end{aligned}$$