Transportation Problem

Promila Kumar

Introduction

The Transportation problem is one of the sub-classes of Linear programming problem in which the objective is to transport various amounts of a single homogeneous commodity, that are initially stored at various origins to different destinations in such a way tha the transportation cost is minimum

Mathematical Formulation

- Let $O_1, O_2, \dots O_n$ $be\ n\ origins$
- Let $D_1, D_2, \dots D_m$ $be\ m\ destinations$
- Let $a_1, a_2, \dots a_n$ be $the\ amount\ of\ comodity\ available\ at\ various\ origins$
- Let $b_1, b_2, \dots b_m$ $be\ the\ amount$ $of\ comodity\ required\ at\ various\ destination$
- Let t_{ij} $be\ the\ cost\ of\ transporting\ one\ unit$ $from\ origin\ O_i\ to\ destination\ D_j$

Transportation Table

Origins/destinations	D_1	D_2			D_n	Availability
O_1	xij c_{11} t_{11}	c_{12} t_{12}	-	-	c_{1n} t_{1n}	a_1
O_2	c_{21} t_{21}	c_{22} t_{22}	xij -	xij -	c_{2n} t_{2n}	a_2
-	xij -	xij -	-	-	-	-
-	-	-	-	- xij	-	-
O_m	c_{m1} t_{m1}	c_{m2} t_{m2}	-	-	c_{mn} t_{mn}	a_m
Requirement	b_1	b_2	-	-	b_m	$\sum_{i=1}^{m} a_{ij} = \sum_{j=1}^{n} b_{ij}$

Transportation Algorithm

1. Obtain the basic feasible solution.
2. Investigate current solution for optimality
   1. If the current solution is optimal, stop.
   2. If not, use optimality conditions of Simplex Method to determine the entering variable from non basic variables.
3. Use feasibility conditions of Simplex Method to determine leaving variable from the set of basic variables and find new basic feasible solution.
4. Go to step 2.

Basic Feasible Solution

- North West Corner Rule
- Least Cost Method
- Vogles Approximation Method

North West Corner Rule

1. Allocate maximum number of units possible in the top left corner of the transportation table.
2. Rewrite demand and availability variables.
3. Eliminate first column if demand is exhausted and first row if requirement is exhausted.
4. Repeat step 1 till all requirements are met.

Least Cost Method

1. Identify the cell with lowest cost.
2. Allocate maximum number of units possible in that cell.
3. Rewrite demand and availability variables.
4. Repeat step 1 till all requirements are met.

1.Obtain Basic Feasible solution for the following problem1

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Requirement	4	6	8	6	24

$$Here \quad \sum_{i=1}^{m} a_{ij} = \sum_{j=1}^{n} b_{ij} = 24 \text{ so it is a balanced transportation problem}$$

Table 1

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	1	2	3	4	6
O_2	4	3	2	6 (allocated), 0	8(2)
O_3	0	2	2	1	10
Requirement	4	6	8	6(0)	24

Least cost is in cell O_2 D_4 and in O_3 D_1 ,so let us select O_2 D_4 as it can have more allocation than O_3 D_1. After allocating 6 units to this cell, demand for D_4 is fulfilled.Delete this and work on the remaining table.

Table 2

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	1	2	3	4	6
O_2	4	3	2	0 (6)	(8)2
O_3	0 (4)	2	2	1	(10)6
Requirement	(4)0	6	8	6(0)	24

Now Least cost is in cell O_3 D_1 ,so let us allocate 4 units to this cell. Demand of destination D_1 is fulfilled. Delete this column and work on the remaining table.

Table 3

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	1	6 2	3	4	(6)0
O_2	4	3	2	6 0	(8)2
O_3	4 0	2	2	1	(10)6
Requirement	(4)0	(6)0	8	6(0)	24

Now Least cost is in cell O_1 D_2 ,so let us allocate 6 units to this cell. Demand of destination D_1 and origin O_1 is fulfilled. Delete this column as well as row and work on the remaining table.

Table 4

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	1	6 2	3	4	(6)0
O_2	4	3	2 2	6 0	(8)(2)0
O_3	4 0	2	6 2	1	(10)(6)0
Requirement	(4)0	(6)0	(8)0	6(0)	24

Now The only possibility left is shown above. Basic feasible solution is obtained. It has (4+3-1=6)5 basic cells, so the solution is degenerate and the cost is 6.2+2.2+6.0+4.0+2.6

2.Obtain Basic Feasible solution for the following problem

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	13	11
O_2	17	18	14	23	13
O_3	32	27	18	41	19
Requirement	6	10	12	15	43

$$Here \quad \sum_{i=1}^{m} a_{ij} = \sum_{j=1}^{n} b_{ij} = 43 \text{ so it is a balanced transportation problem}$$

Table 1

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	(11) 13	(11)0
O_2	17	18	14	23	13
O_3	32	27	18	41	19
Requirement	6	10	12	(15)4	43

Now Least cost is in cell O_1 D_4 ,so let us allocate 11 units to this cell. Stock of Origin O_1 is exhausted. Delete this row and work on the remaining table.

Table 2

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	13 (11)	(11)0
O_2	17	18	14 (12)	23	(13)1
O_3	32	27	18	41	19
Requirement	6	10	(12)0	(15)4	43

Now Least cost is in cell O_2 D_3 ,so let us allocate 12 units to this cell. Demand of destination D_3 is fulfilled. Delete this column and work on the remaining table.

Table 3

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	11 / 13	(11)0
O_2	1 / 17	18	12 / 14	23	(13)(1)0
O_3	32	27	18	41	19
Requirement	(6)5	10	(12)0	(15)4	43

Now Least cost is in cell O_2 D_1 ,so let us allocate 12 units to this cell. Stock of Origin O_2 is exhausted. Delete this row and work on the remaining table.

Table 4

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	(11) 13	(11)0
O_2	(1) 17	18	(12) 14	23	(13)(1)0
O_3	32	(10) 27	18	41	(19)9
Requirement	(6)5	(10)0	(12)0	(15)4	43

Now Least cost is in cell O_3 D_2 ,so let us allocate 10 units to this cell. Demand of Origin D_2 is fulfilled. Delete this column and work on the remaining table.

Q3.Obtain Basic Feasible solution for the following problem1

Origins/destinations	D_1	D_2	D_3	Availability
O_1	13	15	16	17
O_2	7	11	2	12
O_3	19	20	9	16
Requirement	14	8	23	

$$Here \quad \sum_{i=1}^{m} a_{ij} = \sum_{j=1}^{n} b_{ij} = 45 \text{ so it is a balanced transportation problem}$$

Here is basic feasible solution to the problem

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	(11) 13	(11)0
O_2	(1) 17	18	(12) 14	23	(13)(1)0
O_3	(5) 32	(10) 27	18	(4) 41	(19)(9)0
Requirement	(6)(5)0	(10)0	(12)0	(15)(4)0	43

Now The only possibility left is shown above. Basic feasible solution is obtained. It has 4+3-1=6 basic cells, so the solution is non degenerate and the cost is 17.1+14.2+13.11+32.5+27.10+41.4

Vogel's Approximation Method

- For each row identify lowest and next to lowest cost and find their difference and write it at the extreme right.
- Repeat the same for all the columns and write it at the bottom.
- Select a row or column having the maximum of this difference.
- In the selected row or column identify a cell with the minimum cost.
- Allocate the maximum possible number in that cell.
- Adjust the rim requirements and repeat the process.

Q3. Table 1

Origins/destinations	D_1	D_2	D_3	Availability
O_1	13	15	16	17
O_2	7	11	2	12
O_3	19	20	9	16
Requirement	14	8	23	

Now Least cost is in cell O_3 D_3 ,so let us allocate 16 units to this cell. Demand of destination O_3 is fulfilled. Delete this row and work on the remaining table.

Q3. Table 1

Origins/destinations	D_1	D_2	D_3	Availability	Difference
O_1	13	15	16	17	2
O_2	7	11	2	12	5
O_3	19	20	16 / 9	(16)0	10
Requirement	14	8	(23)7		
Difference	6	4	7		

Now Least cost is in cell O_3 D_3 ,so let us allocate 16 units to this cell. Demand of destination O_3 is fulfilled. Delete this row and work on the remaining table.

Q3 Table 2

Origins/destinations	D_1	D_2	D_3	Availability	Difference
O_1	13	15	16	17	2
O_2	7	11	(7) 2	(12)5	5
O_3	19	20	(16) 9	(16)0	10
Requirement	14	8	(23)(7)0		
Difference	6	4	14		

Now Least cost is in cell O_2 D_3 ,so let us allocate 7 units to this cell. Demand at destination D_3 is fulfilled. Delete this column and repeat the process on the remaining table.

Q3 Table 3

Origins/destinations	D_1	D_2	D_3	Availability	Difference
O_1	13	15	16	17	2
O_2	5 · 7	11	7 · 2	(12)(5)0	4
O_3	19	20	16 · 9	(16)0	
Requirement	(14)9	8	(23)(7)0		
Difference	6	4			

Now Least cost is in cell O_2 D_1 ,so let us allocate 5 units to this cell. Stock of Origin O_2 is exhausted. Delete this row and repeat the process on the remaining table.

Q3 Table 4

Origins/destinations	D_1	D_2	D_3	Availability	Difference
O_1	9 13	8 15	16	(17)0	
O_2	5 7	11	7 2	(12)(5)0	
O_3	19	20	16 9	(16)0	
Requirement	(14)(9)0	(8)0	(23)(7)0		
Difference					

Now The only possibility left is shown above. Basic feasible solution is obtained. It has 3+3-1=5 basic cells, so the solution is non degenerate and the cost is 13.9+15.8+5.7+2.7+16.9

Q4.Obtain Basic Feasible solution for the following problem

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	10	10	16	20	300
O_2	16	6	17	25	200
O_3	8	21	10	15	250
Requirement	325	175	100	150	750

$$Here \quad \sum_{i=1}^{m} a_{ij} = \sum_{j=1}^{n} b_{ij} = 750 \text{ so it is a balanced transportation problem}$$

Q4.Obtain Basic Feasible solution for the following problem

Origins/destinations	D_1	D_2	D_3	D_4	Availability	
O_1	10	10	16	20	300	0
O_2	16	6 (175)	17	25	(200)25	10
O_3	8	21	10	15	250	2
Requirement	325	(175)0	100	150	750	
	2	4	6	5		

Now Least cost is in cell O_2 D_2 ,so let us allocate 175 units to this cell. Demand at destination D_2 is fulfilled. Delete this column and repeat the process on the remaining table.

Q4.Table 2

Origins/destinations	D_1	D_2	D_3	D_4	Availability	
O_1	10 (300)	10	16	20	(300)0	6
O_2	16	6 (175)	17	25	(200)25	1
O_3	8	21	10	15	250	2
Requirement	(325)25	(175)0	100	150	750	
	2		6	5		

Now Least cost is in cell O_1 D_1 ,so let us allocate 300 units to this cell. Stock of Origin O_1 is exhausted. Delete this row and repeat the process on the remaining table.

Q4. Table 3

Origins/destinations	D_1	D_2	D_3	D_4	Availability	
O_1	10 (300)	10	16	20	(300)0	
O_2	16	6 (175)	17	25	(200)25	1
O_3	8	21	10	15 (150)	(250)100	2
Requirement	(325)25	(175)0	100	(150)0	750	
	8		7	10		

Now Least cost is in cell O_3 D_4 ,so let us allocate 150 units to this cell. Demand at destination D_4 is fulfilled. Delete this column and repeat the process on the remaining table.

Q4. Table 4

Origins/destinations	D_1	D_2	D_3	D_4	Availability	
O_1	10 (300)	10	16	20	(300)0	
O_2	16	6 (175)	17	25	(200)25	1
O_3	8 (25)	21	10	15 (150)	(250)(100)75	2
Requirement	(325)(25)0	(175)0	100	(150)0	750	
	8		7			

Now Least cost is in cell O_3 D_1 ,so let us allocate 25 units to this cell. Demand at destination D_1 is fulfilled. Delete this column and repeat the process on the remaining table.

Q4. Table 5

Origins/destinations	D_1	D_2	D_3	D_4	Availability	
O_1	10 (300)	10	16	20	(300)0	
O_2	16	6 (175)	17 (25)	25	(200)(25)0	
O_3	8 (25)	21	10 (75)	15 (150)	(250)(100)(75)0	
Requirement	(325)(25)0	(175)0	(100)0	(150)0	750	

Now The only possibility left is shown above. Basic feasible solution is obtained. It has 4+3-1=6 basic cells, so the solution is non degenerate and the cost is 300.10+175.6+17.25+25.8+10.75+150.15

.

Moving Towards Optimality

- Stepping stone method

- Modified Distribution Method or MODI Method

Stepping Stone Method

- Determine an initial basic feasible solution.
- Make sure that number of occupied(basic) cells is exactly m+n-1.
- Evaluate cost effectiveness of shipping one unit of good via transportation routes which are not currently in the solution by following five steps as follows:

1) Select an unoccupied cell.
2) Beginning at this cell trace a closed path.
3) Assign plus(+) and minus(-) sign alternatively on each corner cell of the closed path. The cells at the corner points are called stepping stones.
4) Compute the net change in the cost along the closed path.
5) If net change corresponding to each unoccupied cell is positive, the current solution is optimal, if not…
6) Select the unoccupied cell having the highest negative net cost change and determine the maximum number of units that can be assigned to this cell.
7) Adjust the value of other cells in the closed path.
8) Go to step 1 and repeat the procedure.

Q5. Initial basic feasible is given and apply stepping stone method to find the optimal solution

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	13 (11)	11
O_2	17 (1)	18	14 (12)	23	13
O_3	32 (5)	27 (10)	18	41 (4)	19
Requirement	6	10	12	15	43

Now the total transportation cost is
11.13+1.17+12.14+32.5+27.10+4.41=922
Total number of basic cells is 4+3-1=6

Table 2: select a non basic cell O_1D_1 and make closed path

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21 +	16	25	13 - 11	11
O_2	17 1	18	14 12	23	13
O_3	32 - 5	27 10	18	41 + 4	19
Requirement	6	10	12	15	43

Now the total cost change will be 21 – 13 + 41 – 32= 17

Table 3: select a non basic cell O_1D_2 and make closed path

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16 +	25	13 11 -	11
O_2	17 1	18	14 12	23	13
O_3	32 5	27 10 -	18	41 4 +	19
Requirement	6	10	12	15	43

Now the total cost change will be 16 – 13 + 41 – 27= 17

Table 4: select a non basic cell O_1D_3 and make closed path

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25 (+)	13, 11 (-)	11
O_2	17, 1 (+)	18	14, 12 (-)	23	13
O_3	32, 5 (-)	27, 10	18	41, 4 (+)	19
Requirement	6	10	12	15	43

Now the total cost change will be 25 – 13 + 41 – 32 +17 - 14= 24

Table 5: select a non basic cell O_2D_2 and make closed path

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	13 (11)	11
O_2	17 - (1)	18 +	14 (12)	23	13
O_3	32 + (5)	27 - (10)	18	41 (4)	19
Requirement	6	10	12	15	43

Now the total cost change will be 18 – 27 + 32 – 17=6

Table 6: select a non basic cell O_2D_4 and make closed path

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	13 (11)	11
O_2	17 − (1)	18	14 (12)	23 +	13
O_3	32 + (5)	27 (10)	18	41 − (4)	19
Requirement	6	10	12	15	43

Now the total cost change will be 23 – 41 + 32 – 17= -3

Table 7: select a non basic cell O_3D_3 and make closed path

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	13 (11)	11
O_2	17 + (1)	18	14 - (12)	23	13
O_3	32 - (5)	27 (10)	18 +	41 (4)	19
Requirement	6	10	12	15	43

Now the total cost change will be 18 – 14 +17 – 32= - 11 which is most negative , so this cell will become occupied or basic cell. Maximum allocation that can be made in this cell is 5. Hence now O_3 D_1 will become non basic cell. The new basic solution will look like:

Instead of making all these tables we can make 1 table as follows

Unoccupied Cell	Closed Path			Net cost change
$O_1 D_1$ →	$O_1 D_4$ →	$O_3 D_4$ →	$O_3 D_1$	21 – 13 + 41 – 32= 17
$O_1 D_2$ →	$O_1 D_4$ →	$O_3 D_4$ →	$O_3 D_2$	16 – 13 + 41 – 27= 17
$O_1 D_3$ →	$O_1 D_4$ →	$O_3 D_4$ →	$O_3 D_1$ → | $O_2 D_1$ → | $O_2 D_3$	25 – 13 + 41 – 32+ 17-14=24
$O_2 D_2$ →	$O_3 D_2$ →	$O_3 D_1$ →	$O_2 D_1$	18 – 27 + 32 – 17= 6
$O_2 D_4$ →	$O_3 D_4$ →	$O_3 D_1$ →	$O_2 D_1$	23 – 41 + 32 – 17= - 3
$O_3 D_3$ →	$O_3 D_1$ →	$O_2 D_1$ →	$O_2 D_3$	18 – 32 + 17 – 14= - 11
The most negative net cost change is – 11 corresponding to O_3 D_3. So this cell will now become a basic cell.				

Table 8: New improved basic feasible solution

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	13 11	11
O_2	17 6	18	14 7	23	13
O_3	32	27 10	18 5	41 4	19
Requirement	6	10	12	15	43

Now the transportation cost will be

17.6 + 14.7 + 13.11 + 27.10 +18.5 + 41.4 = 867 which is definitely better.

We repeat the whole process

Stepping Stone Method (table 8)

Unoccupied Cell	Closed Path					Net cost change
$O_1 D_1$ →	$O_1 D_4$ →	$O_4 D_3$ →	$O_3 D_3$ →	$O_2 D_3$ →	$O_2 D_1$	21 – 13 + 41 – 18 + 14 - 17=21
$O_1 D_2$ →	$O_1 D_4$ →	$O_3 D_4$ →	$O_3 D_2$			16 – 13 + 41 – 27= 17
$O_1 D_3$ →	$O_1 D_4$ →	$O_3 D_4$ →	$O_3 D_3$			25 – 13 + 41 – 18 = 35
$O_2 D_2$ →	$O_2 D_3$ →	$O_3 D_3$ →	$O_3 D_2$			18 – 14 + 18 – 27= -5
$O_2 D_4$ →	$O_2 D_3$ →	$O_3 D_3$ →	$O_3 D_4$			23 – 14 + 18 – 41= - 14
$O_3 D_1$ →	$O_3 D_3$ →	$O_2 D_3$ →	$O_2 D_1$			32 – 18 + 14 – 17= 11

The most negative net cost change is – 14 corresponding to O_2 D_4. So this cell will now become a basic cell.

Table 9: New improved basic feasible solution

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	13 (11)	11
O_2	17 (6)	18	14 (3)	23 (4)	13
O_3	32	27 (10)	18 (9)	41	19
Requirement	6	10	12	15	43

Now the transportation cost will be
17.6 + 14.3 + 13.11 + 27.10 +18.9 +23.4 = 811 which is definitely better.
We repeat the whole process

Stepping Stone Method (table 9)

Unoccupied Cell	Closed Path	Net cost change
$O_1 D_1$ →	$O_1 D_4$ → $O_2 D_4$ → $O_2 D_1$	21 – 13 + 23 – 17 =14
$O_1 D_2$ →	$O_1 D_4$ → $O_2 D_4$ → $O_2 D_3$ → $O_3 D_3$ → $O_3 D_2$	16 – 13 + 23 – 14 +18 - 27= 3
$O_1 D_3$ →	$O_1 D_4$ → $O_2 D_4$ → $O_2 D_3$	25 – 13 + 23 – 14 = 21
$O_2 D_2$ →	$O_2 D_3$ → $O_3 D_3$ → $O_3 D_2$	18 – 14 + 18 – 27= -5
$O_3 D_4$ →	$O_2 D_4$ → $O_2 D_3$ → $O_2 D_1$	41 – 23 + 14 – 18= 14
$O_3 D_1$ →	$O_3 D_3$ → $O_2 D_3$ → $O_2 D_1$	32 – 18 + 14 – 17= 11

The most negative net cost change is – 5 corresponding to O_2 D_2. So this cell will now become a basic cell.

Table 10: New improved basic feasible solution

Origins/destinations	D_1	D_2	D_3	D_4	Availability
O_1	21	16	25	13 (11)	11
O_2	17 (6)	18 (3)	14	23 (4)	13
O_3	32	27 (7)	18 (12)	41	19
Requirement	6	10	12	15	43

Now the transportation cost will be
17.6 + 18.3 + 13.11 + 27.7 +18.12 +23.4 = 796 which is definitely better.
We repeat the whole process

Stepping Stone Method (table 10)

Unoccupied Cell	Closed Path				Net cost change
$O_1 D_1$ →	$O_1 D_4$ →	$O_2 D_4$ →	$O_2 D_1$		21 − 13 + 23 − 17 =14
$O_1 D_2$ →	$O_1 D_4$ →	$O_2 D_4$ →	$O_2 D_2$		16 − 13 + 23 − 18 = 8
$O_1 D_3$ →	$O_1 D_4$ →	$O_2 D_4$ →	$O_2 D_3$ →	→	25 − 13 + 23 − 14 = 26
$O_2 D_3$ →	$O_2 D_3$ →	$O_3 D_3$ →	$O_3 D_2$		18 − 14 + 18 − 27= 5
$O_3 D_4$ →	$O_2 D_4$ →	$O_2 D_3$ →	$O_2 D_1$		41 − 23 + 14 − 18= 9
$O_3 D_1$ →	$O_3 D_3$ →	$O_2 D_3$ →	$O_2 D_1$		32 − 18 + 14 − 17= 6

The (net change) opportunity cost corresponding to each unoccupied cell is positive the current table gives the optimal solution.

Moving Towards Optimality

- Detemine an initial basic feasible solution consisting of m + n – 1 basic variables.
- Determine set of numbers u_i (i=1,2….,m) for each row and v_j (j=1,2,…,n) for each column so that $C_{i,j} = u_i + v_j$ for basic(occupied) cells.
- The process can be initiated by assigning value ‘0’ to any one of these u_i or v_j.

- For all non-basic cells compute $u_i + v_j - c_{i,j}$
- Enter them on one corner of the corresponding cell.
- If all $u_i + v_j - c_{i,j} \leq 0$ then the current solution is optimal.
- If at least one $u_i + v_j - c_{i,j} > 0$ select the one with the largest value to become new basic variable.
- Make a loop starting from the selected cell and assign values $+\theta$ and $-\theta$ alternatively.
- Assign maximum value to θ so that one basic variable becomes zero and others remain positive.
- Repeat the process till all $u_i + v_j - c_{i,j} \leq 0$.

Q6. Find the optimal solution for the given problem:

Origins/destinations	D_1	D_2	D_3	D_4	Availability	
O_1	5	3 (18)	6 (1)	2	19	$u_1 = 0$
O_2	4	7	9 (12)	1 (25)	37	$u_2 = 3$
O_3	3 (16)	4	7 (18)	5	34	$u_3 = 1$
Requirement	16	18	31	25	90	
	$v_1 = 2$	$v_2 = 3$	$v_3 = 6$	$v_4 = -2$		

The transportation cost will be

18.3 + 6.1 + 12.9 + 25.1 +16.3 +18.7 = 367

We proceed towards optimality.

Calculating net evaluations

Origins/destinations	D_1	D_2	D_3	D_4	Availability	
O_1	5 2-5=-3	3 18	6 1	2 -2+0-2=-4	19	u_1 =0
O_2	4 3+2-4=1	7 3+3-7=-1	9 12	1 25	37	u_2= 3
O_3	3 16	4 1+3-4=0	7 18	5 1-2-5=-6	34	u_3= 1
Requirement	16	18	31	25	90	
	v_1= 2	v_2= 3	v_3= 6	v_4= -2		

For non basic cells calculate $u_i + v_j - c_{ij}$,
O_2D_1 being largest positive cell will be new basic cell.

Next better solution

Origins/destinations	D_1	D_2	D_3	D_4	Availability	
O_1	5 2-5=-3	3 18	6 1	2 -2+0-2=-4	19	$u_1 = 0$
O_2	4 $+\theta$ 3+2-4=1	7 3+3-7=-1	9 $12-\theta$	1 25	37	$u_2 = 3$
O_3	3 $16-\theta$	4 1+3-4=0	7 $18+\theta$	5 1-2-5=-6	34	$u_3 = 1$
Requirement	16	18	31	25	90	
	$v_1 = 2$	$v_2 = 3$	$v_3 = 6$	$v_4 = -2$		

Assign value θ to cell O_2D_1 and make a loop using basic cells.
Choose θ in such a way that one cell becomes non basic(θ = 12)

Next better solution

Origins/destinations	D_1	D_2	D_3	D_4	Availability	
O_1	5 -3	3 18	6 1	2 -3	19	$u_1 = 0$
O_2	4 12	7 -2	9 -1	1 25	37	$u_2 = 2$
O_3	3 4	4 0	7 30	5 -5	34	$u_3 = 1$
Requirement	16	18	31	25	90	
	$v_1 = 2$	$v_2 = 3$	$v_3 = 6$	$v_4 = -1$		

For all non basic cells calculate $u_i + v_j - c_{ij} \leq 0$,so the current solution is optimal and optimal value is 18.3+6.1+12.4+25.1+4.3+30.7=355