Number Conversions

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INTRODUCTION

Number System – A number system defines as a set of values to represent quantity. Like we talk about the number of people attending a class, the number of modules taken by each student and use numbers to represent grade.

Number system can be categorized into two systems:

(a) Non-Positional Number System
(b) Positional Number System

Non-Positional Number System – In ancient times, people used to count on fingers, when the fingers became insufficient for counting, then people used stones, pebbles or sticks to indicate values. But it was very difficult to perform arithmetic with such a number system as there is no symbol for zero.

Positional Number System – In this system the value of each digit is defined not by the symbol but also by the symbol position. Positional Number System is used to perform arithmetic. Existing Positional Number System is decimal number system. Apart from the decimal number system, there are binary number system, octal number system and hexadecimal number system and so on.

Base (Radix) – In any number system the base or radix tells the number of symbols used in the system. In the earlier days, different civilizations were using different radixes. The Egyptian used the radix 2, the Babylonians used the radix 60 and Mayans used 18 and 20.

The base of a number system is indicated by a subscript (decimal number) and this will be followed by the value of the number. For example $(952)_{10}$, $(456)_8$, $(314)_{16}$

Number System that are used widely by the Computer System are:

Decimal System
Binary System
Octal System
Hexadecimal System

Decimal System –The number system includes the ten digits from 0 to 9. These digits are recognized as the symbols of the decimal system. Each digit is with base 10 and n^{th} digit of the number has a power of $(10)^{n-1}$.

For example-
$9542 = 9000 + 500 + 40 + 2 = (9 \times 10^3) + (5 \times 10^2) + (4 \times 10) + (2 \times 10^0)$

Binary system – Computers do not use the decimal system for counting and arithmetic. Their CPU and memory are made up of millions of tiny switches that can be in either ON or OFF states. 0 represents OFF and 1 represents ON. In this way we use binary system in a computer.

Binary system has two numbers 0 and 1. Binary system has base 2 therefore the weight of n^{th} bit of the number nth bit x 2^{n-1}.

Octal System – The octal system is commonly used by computers. The octal number system includes 8 digits 0, 1, 2, 3, 4, 5, 6, 7 and has base 8. The octal system uses a power of 8 to determine the digit of a number's position.

Hexadecimal System – Hexadecimal is another number system that works exactly like the decimal, binary and octal number systems, except that has the base 16. Each hexadecimal represents a power of 16. The system uses 0 to 9 numbers and A to F characters to represent 10 to 15 respectively.

Conversions – Any number system can be converted into another number system. There are various methods that can be used in converting numbers from one base to another.

Conversions of Decimal to Binary – The method that is used for converting decimals into binary is known as the remainder method. We use the following steps in getting the binary number-

(a) Divide the decimal number by 2.
(b) Write the remainder (which is either 0 or 1) at the right most position.
(c) Repeat this process of dividing it by 2 until the quotient is 0 and keep writing the remainder after each step of division.
(d) Write the remainders in reverse order.

Example – Convert $(45)_{10}$ into binary number system.

2	45	Remainder
2	22	1
2	11	0
2	5	1
2	2	1
2	1	0
	0	1

Thus $(45)_{10} = (101101)_2$

Note – In every number system-

(a) The first bit from the right is referred as LSB (Least Significant Bit)
(b) The first bit from the left is referred as MSB (Most Significant Bit)

Conversions of Decimal Fraction to Binary Fraction – For converting decimal fraction into binary fraction, we use multiplication and write the integral part of the outcome.The following steps are used to get the binary fractions-

(a) Multiply the decimal fraction by 2.
(b) If a non-zero integer is generated, record the non-zero integer otherwise record 0.
(c) Remove the non-zero integer and repeat the above steps till the fraction value become 0.
(d) Write down the number according to the occurrence.

Example – Find the binary equivalent of $(0.75)_{10}$.

	Number (to be recorded)
0.75 x 2 = 1.50	1
0.50 x 2 = 1.00	1

Thus $(0.75)_{10} = (0.11)_2$.
Moreover, we can write $(45.75)_{10} = (101101.11)_2$.

Remark – If the conversion is not ended and till continuing; we write the approximation in 16 bits.

Example- Find the binary equivalent of $(0.9)_{10}$.

	Number (to be recorded)
0.9 x 2= 1.8	1
0.8 x 2= 1.6	1
0.6 x 2= 1.2	1
0.2 x 2= 0.4	0
0.4 x 2= 0.8	0
0.8 x 2= 1.6	1
0.6 x 2= 1.2	1
0.2 x 2= 0.4	0
0.4 x 2= 0.8	0
0.8 x 2= 1.6	1
0.6 x 2= 1.2	1
0.2 x 2= 0.4	0
0.4 x 2= 0.8	0
0.8 x 2= 1.6	1
0.6 x 2= 1.2	1
0.2 x 2= 0.4	0
0.4 x 2= 0.8	0
0.8 x 2= 1.6	1

Thus $(0.9)_{10}= (0.111001100110011001)_2$.

Conversion of Decimal to Octal – The conversion of decimal to octal is similar to decimal to binary. Instead of dividing the number by 2, we divide the number by 8.

Example - Convert $(45)_{10}$ into octal number system.

8	45	Remainder
8	5	5
8	0	5

Thus $(45)_{10} = (55)_{8}$.

Conversions of Decimal Fractions to Octal Fractions – The conversion of decimal fraction to octal fraction is similar to decimal fraction to binary fraction. Here we multiply the fraction by 8 instead of 2.

Example – Find the octal equivalent of $(0.75)_{10.}$

Number (to be recorded)
$0.75 \times 8 = 6.00$
Thus $(0.75)_{10} = (0.6)_{8}$.
And $(45.75)_{10} = (55.6)_{8}$.

Conversion of Decimal to Hexadecimal - We divide by 16 instead of 2 or 8. If the remainder is in between 10 to 16, then the number is represented by A to F respectively.

Example – Convert $45)_{10}$ into hexadecimal.

16	45	Remainder
16	2	D
16	0	2

Thus $(45)_{10} = (2D)_{16}$.

Conversions of Decimal Fractions to Hexadecimal Fractions - Here we multiply the fraction by 16 instead of 2 or 8. If the non-zero integer is in between 10 to 16, then the number is represented by A to F respectively.

Example- Find the hexadecimal equivalent of $(0.75)_{10}$.

	Number (to be recorded)
0.75 x 16 = 12.00	C (12-C)

Thus $(0.75)_{10} = (0.C)_{16}$.

And $(45.75)_{10} = (2D.C)_{16}$.

Conversions of Binary to Decimal - In converting binary to decimal, we use the following steps-

(a) Write the weight of each bit.
(b) Get the weighted value by multiplying the weighted position with the respective bit.
(c) Add all the weighted value to get the decimal number.

Example – Convert $(101101)_2$ into decimal number system.

Binary Number	1	0	1	1	0	1
Wt. of each bit	2^5	2^4	2^3	2^2	2^1	2^0
Weighted Value	1×2^5	1×2^4	1×2^3	1×2^2	1×2^1	1×2^0
Solved Multiplication	32	0	8	4	0	1

Thus $(101101)_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2 + 1 \times 2^0$.
$= 32 + 0 + 8 + 4 + 0 + 1$
$= 45$

Conversions of Binary Fraction to Decimal Fraction - The conversions of binary fraction to the decimal fraction is similar to conversion of binary numbers to decimal numbers. Here, instead of a decimal point we have a binary point. The exponential expressions (or weight of the bits) of each fractional placeholder is 2^{-1}, 2^{-2}

Example- Convert $(101101.11)_2$ into decimal number system.

Binary Number	1	0	1	1	0	1	1	1
Wt. of each bit	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
Weighted Value	1×2^5	0×2^4	1×2^3	1×2^2	0×2	1×2^0	1×2^{-1}	1×2^{-2}
Solved Multiplication	32	0	8	4	0	1	0.5	0.25

Thus $(101101.11)_2 = 32 + 0 + 8 + 4 + 0 + 1 + 0.5 + 0.25 = 45.75$

Conversions of Binary to Octal – We use the following steps in converting binary to octal-

(a) Break the number into 3-bit sections starting from LSB to MSB
(b) If we do not have sufficient bits in grouping of 3-bits, we add zeros to the left of MSB so that all the groups have proper 3-bit number.
(c) Write the 3-bit binary number to its octal equivalent.

Example – Convert $(101101)_2$ into octal.

Binary Number	101	101
Octal Number	5	5

Thus $(101101)_2 = (55)_8$.

Example – Convert $(1101101)_2$ into octal.

Binary Number	001	101	101
Octal Number	1	5	5

Thus $(1101101)_2 = (155)_8$.

Conversions of Binary Fraction to Octal Fraction – We use the following steps in converting binary fractions to octal fraction-

(d) Break the fraction into 3-bit sections staring from MSB to LSB.
(e) In order to get a complete grouping of 3 bits, we add trailing zeros in LSB.
(f) Write the 3-bit binary number to its octal equivalent.

Example – Convert $(101101.11)_2$ into octal.

Binary Number	101	101	110
Octal Number	5	5	6

Thus $(101101)_2 = (55.6)_8$.

Conversions of Binary to Hexadecimal – We convert binary to hexadecimal in the similar manner as we have converted binary to octal. The only difference is that here, we from the group of 4-bits.

Example- Convert $(101101)_2$ into hexadecimal.

Binary Number	0010	1101
Decimal Number	2	13
Hexadecimal Number	2	D

Thus $(101.101)_2 = (2D)_{16}$.

Conversions of Binary Fraction to Hexadecimal Fraction– The conversion of binary to hexadecimal is similar to the binary to octal. The only difference is that here we form the group of 4 – bits.

Example – Convert $(101101.11)_2$ into hexadecimal.

Binary Number	0010	1101	1100
Decimal number	2	12	12
Hexadecimal Number	2	D	C

Thus $(101101.11)_2 = (2D.C)_{16}$.

Conversion of Octal to Decimal – We follow the same steps of conversion of binary to decimal. The only difference is that here weight of n^{th} bit is 8^{n-1} instead of 2^{n-1}.

Example – Convert $(55)_8$ into decimal number system.

Octal Number	5	5
Wt. of each bit	8^1	8^0
Weighted Value	5×8	5×8^0
Solved Multiplication	40	5

Thus $(55)_8 = 40+5$.

=45

Conversion of Octal fraction to Decimal fraction - The weight of the bit of the fraction placeholder is 8^{-1}l, 8^{-2}………. The conversion of octal fraction to decimal fraction is similar to the conversion of binary fraction to decimal fraction.

Example – Convert $(55.6)_8$ into decimal number system.

Octal Number	5	5	6
Wt. of each bit	8^1	8^0	8^{-1}
Weighted Value	5×8	5×8^0	6×8^{-1}
Solved Multiplication	40	5	0.75

Thus $(55.6)_8 = 40 + 5 + 0.75 = 45.75$

Conversion of Octal to binary - We use the following steps in conversion of octal to binary-

(a) Convert each octal digit into 3-bit binary equivalent.
(b) Combine the 3-bit section by removing the spaces to get the binary number

Example- Convert $(55)^8$ binary.

Octal Number	5	5
Binary Number	101	101

Thus $(55)_8 = (1010101)_2$

Example – Convert $(456)_8$ into binary.

Octal Number	4	5	6
Binary Number	100	101	110

Thus $(456)_8 = (100101110)_2$

Conversion of Octal Fraction to Binary Fraction – In this, the method of conversion is based on the same procedure that we have discussed in conversion of octal to binary.

Example – Convert $(55.6)_8$ into binary.

Octal Number	5	5	6
Binary number	101	101	110

Thus $(55.6)_8 = (101101.11)_2$.

Conversions of Octal to Hexadecimal- The conversion involves the following steps-
(a) Convert each octal digit into 3 - bit binary form.
(b) Combine all the 3-bit binary numbers.
(c) Grouping them in 4-bit binary form by starting from MSB to LSB.
(d) Convert these 4-bit blocks into their hexadecimal symbols.

Example- Convert $(55)_8$ into hexadecimal.

Octal Number	5	5
Binary Number	101	101

Combining the 3-bit binary block, we have 101101.
Grouping them in 4 bit binary form-

Binary Number	0010	1101
Hexadecimal Symbol	2	D

Thus $(55)_8 = (2D)_{16}$.

Conversions of Octal Fraction to Hexadecimal Fraction- The method of conversion is based on the same procedure that we have discussed in conversions of octal to hexadecimal.

Example- Convert $(55.6)_8$ into hexadecimal.

Octal Number	5	5	6
Binary Number	101	101	110

Combining the 3-bit binary block, we have 101101.110.
Grouping them in 4 bit binary form-

Binary Number	0010	1101	1100
Hexadecimal Symbol	2	D	C

Thus $(55)_8 = (2D.C)_{16}$..

Conversions of Hexadecimal to Decimal- The conversion of hexadecimal to decimal is similar to binary to decimal. Here weight of n^{th} bit is 16^{n-1} instead of 2^{n-1}.

Example- Convert $(2D)_{16}$ into decimal.

Hexadecimal Number	2	D(=13)
Wt. of each bit	16^1	16^0
Weighted Value	2 x 16	2 x 16^0
Solved Multiplication	32	13

Thus $(2D)_{16}$= 32 + 13 = 45.

Conversions of Hexadecimal Fraction to Decimal Fraction- We do the conversion of hexadecimal fraction to decimal fraction in the similar manner as we have done in the conversion of binary fractions to decimal fractions. Here weight of bit is 16^{-1}, **16^{-2}** ....

Example- Convert $(2D.C)_{16}$ into decimal.

Hexadecimal Number	2	D(=13)	C(=12)
Wt. of each bit	16^{1}	16^0	16^{-1}
Weighted Value	2 x 16	13 x 16^0	13 X 16^{-1}
Solved Multiplication	32	13	0.75

Thus $(2D.C)_{16}$= 32 + 13 + 0.75 = 45.75.

Conversions of Hexadecimal to Binary -We use the following steps-

(a) Convert each hexadecimal digit into its 4-bit binary equivalent.
(b) Combine all the binary numbers.

Example- Convert $(2D)_{16}$ into binary.

Hexadecimal Symbol	2	D(=13)
Binary Number	0010	1101

Thus $(2D)_{16}$ = $(00101101)_2$= $(101101)_2$.

Conversions of Hexadecimal Fraction to Binary Fraction – This method of conversion is based on the same procedure that we have discussed in the conversion of hexadecimal to binary.

Example – Convert (2D.C) $_{16}$ into binary.

Hexadecimal Number	2	D(=13)	C(=12)
Binary Number	0010	1101	1100

Thus $(2D)_{16}$ = $(00101101\text{-}1100)_2$ = $(101101\text{-}11)_2$.

Conversions of Hexadecimal to Octal- We use the following steps for the conversion of hexadecimal to octal.

(1) Convert each hexadecimal digit into binary.
(2) Grouping all the binary numbers into 4-bit form.
(3) Again grouping them into 3-bit form.
(4) Convert the 3-bit block in octal.

Example- Convert $(2D)_{16}$ into octal.

Hexadecimal Number	2	D(=13)
Binary Number	0010	1101

Combining the binary number, we get 00101101=101101
Grouping the binary number in 3-bit

Binary Number	101	101
Octal Number	5	5

Thus $(2D)_{16}$ = $(55)_8$.

Conversions of Hexadecimal Fractions to Octal Fractions – The method of conversion is based on the same procedure that we have discussed in the conversion of hexadecimal to octal.

Example- Convert $(2D.C)_{16}$ into octal.

Hexadecimal Number	2	D(=13)	C(=12)
Binary Number	0010	1101	1100

Combining the binary number, we get 00101101.1100 = 101101.11
Grouping the binary number in 3-bit

Hexadecimal Number	101	101	110
Octal Number	5	5	6

Thus $(2D.C)_{16} = (55.6)_8$