Orientation Programme (OR-95)

27th November 2018 to 24th December 2018

Topic: Numeral System

Submitted as partial fulfilment of UGC sponsored
Orientation Programme

By
Dr.Sarita Agarwal
Department of Mathematics
Acharya Narendra Dev College
University of Delhi
Delhi,India

CENTRE FOR PROFESSIONAL DEVELOPMENT IN HIGHER EDUCATION
UNIVERSITY OF DELHI

DELHI-110007

Declaration of Originality

Name: Dr.Sarita Agarwal

Subject:Mathematics

College:Acharya Narendra Dev College

University: University of Delhi

Title of the paper: Numeral System

I hereby declare that the paper titled “Numeral System” is submitted to fulfill the requirement towards completion of my Orientation Programme (OR-95) (27th November 2018 - 24th December 2018), at CPDHE, University of Delhi, represents my original work and that I have used no other sources except those that I have indicated in the references. All data, tables, figures and text citations which have been reproduced from any other source, including the internet, have been explicitly acknowledged as such. I am aware of the consequences of non-compliance of my above declaration.

Signature:

Place: New Delhi

Date:18.12.2018

NUMERAL SYSTEM

Numeral is a figure, symbol or group of figures or symbols used for expressing numbers. A number system [10] is a set of values to represent quantity.

Numbers are the mathematical tool to count, measure and label. Who invented numbers? Still a question. But each civilisation has a certain set of ways to express numerals. In early ages, people expressed numerals by either the different voices or the different mono act. They even used fingers for counting [10] when fingers became insufficient for counting, stones and pebbles were used to indicate values.

Later when the people had become civilised, they started using symbols for numerals.

Number System can be categorized in two system.

1. <u>Non-Positional Number System</u>[10]

A system in which fingers, stones, pebbles or stick were used to indicate values. But it was very difficult to perform arithmetic with such number system as there was no symbol for zero.

2. <u>Positional Number System</u> [10]

In this system, the value of each digit is defined not only by the symbol but also by the decimal position. Positional Number system is used to perform arithmetic.

Each number system has a certain base or ratio. We are using various number systems in the scientific world. The most common number system are--

1. Binary Number System
2. Octal Number System
3. Decimal Number System
4. Hexadecimal Number System

Binary Number System

To describe prosody, a binary system was developed by Indian scholar Pingala in 2 B.C. He invented Morse code by using binary numbers in the form of short and long syllabus. For that he used **.** (dot) and **–** (dash) .

Morse code [11] is character encoding scheme used in telecommunications that encodes text character as standardised sequences of two different signal duration.

Thomas Harriot, Joan Caramuel Lobkowitz and Gottfried Leibniz studied the modern binary code in the 16th and 17th centuries.

Leibniz studied binary number in 1679; his work appears in his article *Explication de l'Arithmétique Binaire(Explanation of Binary Arithmetic)* which uses only the characters, 1 and 0, like the modern binary numeral system. An example of Leibniz's binary numeral system is as follows:

0 0 0 1   numerical value 2^0
0 0 1 0   numerical value 2^1
0 1 0 0   numerical value 2^2
1 0 0 0   numerical value 2^3.

In 1854, British mathematician George Boole published a landmark paper , detailing an algebraic system of logic that later became Boolean algebra. His logical calculus is instrumental in the design of digital electronic circuits. [11]

In 1937, Boolean algebra and Binary arithmetic using electronic relays and switches was implemented by Claude Shannon in his Master's thesis at MIT. Entitled *A Symbolic Analysis of Relay and Switching Circuits*, Shannon's thesis founded practical digital circuit design. [11]

Bell Labs with Stibitz worked with Complex Number Computer. Their Complex Number Computer, completed on 8 January 1940, was able to calculate the values of complex numbers. In a demonstration to the American Mathematical Society conference at Dartmouth College on 11 September 1940, Stibitz commanded the Complex Number Calculator remote over telephone lines by a teletype. It was the first computing machine ever used over a phone line. Well-known mathematicians John von Neumann, John Mauchly and Norbert Wiener wrote about his demonstration in their memoirs. [11]

Konrad Zuse designed Z1 computer between 1935 and 1938, used Boolean logic and binary floating point numbers.[11]

Digital devices perform mathematical operations in switching on and off the device. On is marked by 1 and off is marked by 0. In this way the calculation of the computer became faster. Logic Gates is a physical device that uses Boolean function in performing logical operation AND,OR,NOT….

Binary Code

In a binary system when numerals are used in a code and that code is called binary code.

[12 **Braille** which is a type of binary code widely used by the blind to read and write. This system consists of grids of six dots each, three per column, in which each dot has two states: raised or not raised. All letters, numbers, and punctuation signs are represented by the different combinations of raised and flattened dots.

Uses of binary codes

- 1875: Émile Baudot "Addition of binary strings in his ciphering system," which, eventually leads to the ASCII of today.
- 1884: The Linotype machine where the matrices are sorted to their corresponding channels after using by a binary-code slide rail.
- 1932: C. E. Wynn-Williams "Scale of Two" counter.
- 1937: Alan Turing electro-mechanical binary multiplier.
- 1937: George Stibitz "excess three" code in the Complex Computer.
- 1937: Atanasoff-Berry Computer .

- 1938: Konrad Zuse Z1.

Now-a-days, binary encoding is used for instructions and data. CDs, DVDs, and Blu-ray Discs represent sound and video digitally in binary form. Telephone calls are carried digitally on long-distance and mobile phone networks using pulse-code modulation, and on

Octal Number System

Octal numeral system is the base-8 number system, and uses the digits 0 to 7.

Dalia Raut (2012) in his paper, "A Brief History of Indian Classical Music" has written that amongst the four Vedic scriptures, "Samved" was primarily the music based. The matras were known as **Samgan** in **Samved** and had only three types of tones, which were Anudatta(Low pitch) ,Udataya(High pitch) and Swarit(between low and high pitch).

The age of Ramayana and Mahabharata is considered to be the golden age of Gandarbha music. It is believed that during this era , the seven swars (Sa, Re, Ga, Ma, Pa, Dha, Ni) were developed.

If we observe carefully, we get that Indian classical music starts with Sa and ends with Sa. The table with notes, pitches and numbers are as follows---

Swar $\longrightarrow$	sa	re	ga	Ma	pa	dha	ni	sa
Pitch	f^0	f^1	f^2	f^3	f^4	f^5	f^6	f^7
Number	0	1	2	3	4	5	6	7
	First							Last

Indian Classical music lies within 8 notes. This indicates that they were aware of octal code.

Even the Yuki language in California and the Pamean languages in Mexico [13] have octal systems as the speakers count using the spaces between their fingers.

In 1668, John Wilkins proposed the use of base 8 instead of 10 in "*An Essay towards a Real Character, and a Philosophical Language*"

"In 1716 King Charles XII of Sweden[13] asked Emanuel Swedenborg to elaborate a number system based on 64 instead of 10. Swedenborg however argued that base 64 would be difficult for people with less intelligence so he proposed 8 as the base. In 1718 Swedenborg wrote (but did not publish) a manuscript: "En ny rekenkonst som om vexlas wid Thalet 8 i stelle then wanliga wid Thalet 10" which means "A new arithmetic (or art of counting) which changes at the Number 8 instead of the usual at the Number System 10". The numbers 1-7 are there denoted by the consonants l, s, n, m, t, f, u (v) and zero by the vowel o. Thus 8 = "lo", 16 = "so", 24 = "no", 64 = "loo", 512 = "looo" etc. Numbers with consecutive consonants are pronounced with vowel sounds between in accordance with a special rule."

(London) July 1745, Hugh Jones proposed an octal system for British coins, weights and measures. "Whereas reason and convenience indicate us an uniform standard for all quantities; which we call the *Georgian standard*; and that is only to divide every integer in each *species* into eight equal parts, and every part again divided into 8 real or imaginary particles, as far as necessary. 8 is a far more complete and commodious number; since it is divisible into halves, quarters, and half quarters (or units) without a fraction, while subdivision of *ten* is incapable...." In a later treatise on Octave computation (1753) Jones concluded: "Arithmetic by *Octaves*seems most agreeable to the Nature of Things, and therefore may be called Natural Arithmetic in Opposition to that now in use by decades, which may be esteemed Artificial Arithmetic."

In 1801, James Anderson criticized the French for basing the Metric system on decimal arithmetic. He suggested base 8, for which he coined the term *octal*. Through his work, he noticed that the system of English units was already, to a remarkable extent, an octal system.

In the mid-19th century, Alfred B. Taylor concluded that "Our octonary [base 8] radix is, therefore, beyond all comparison the "*best possible one*" for an arithmetical system." A graphical notation for the digits and new names for the numbers was a part of the proposal presented, prompting that we should count "*un*, *du*, *the*, *fo*, *pa*, *se*, *ki*, *unty*, *unty-un*, *unty-du*" and so on, with successive multiples of 8 named "*unty*, *duty*, *thety*, *foty*, *paty*, *sety*, *kity* and *under*." So, for example, the number 65 (101 in octal) would be spoken in octonary as *under-un*. Some of Swedenborg's work on octonary were also republished by Taylor.

In computers [13]

Octal became widely used in computer systems such as the PDP-8, ICL 1900 and IBM mainframes employed 12-bit, 24-bit or 36-bit words. Octal was an ideal abbreviation of binary for these machines because of their word size is divisible by three (each octal digit represents three binary digits).

All modern computing platforms, however, use 16-, 32-, or 64-bit words, further divided into eight-bit bytes. On such systems, three octal digits per byte would be required, with the most significant octal digit representing two binary digits (plus one bit of the next significant byte, if any). Octal representation of a 16-bit word requires 6 digits, but the most significant octal digit represents (quite inelegantly) only one bit (0 or 1).

Octal is sometimes used in computing instead of hexadecimal, perhaps most often in modern times in conjunction with file permissions under Unix systems . It has the advantage of not requiring any extra symbols as digits.

Decimal Number System

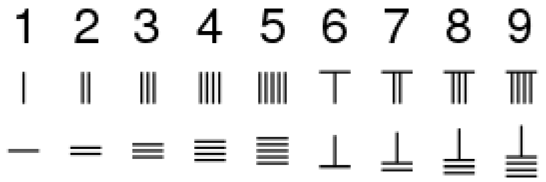
Decimal number system is the base-10 number system and uses digit 0 to 9.

Many ancient cultures[15] calculated numerals with base ten, sometimes chaos happened because human hands typically having ten digits.

Standardized weights used in Indus Valley Civilization(c.3300-1300 BCE) were based on the ratios: 1/20, 1/10, 1/5, 1/2, 1, 2, 5, 10, 20, 50, 100, 200, and 500, while their standardized ruler – the *Mohenjo-daro ruler* – was divided into ten equal parts.

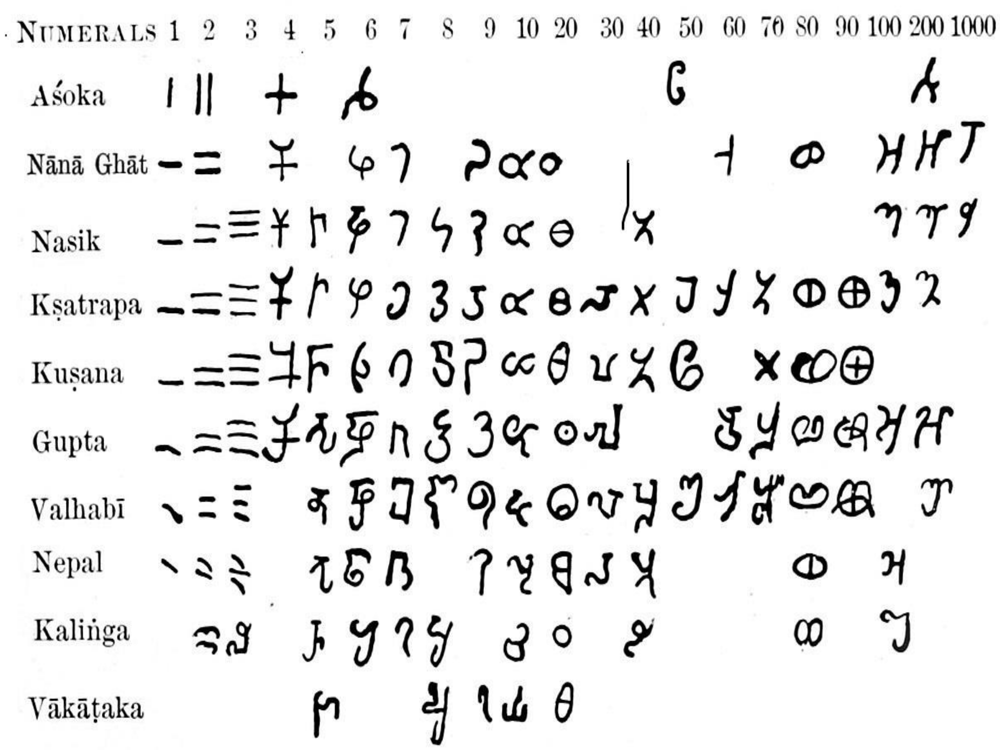
Egyptian hieroglyphs, in evidence since around 3000 BC, used a purely decimal system, just as the Cretan hieroglyphs (ca. 1625−1500 BC) of the Minoans whose numerals are closely based on the Egyptian model. The decimal system was handed down to the consecutive Bronze Age cultures of Greece, including Linear A (ca. 18th century BC−1450 BC) and Linear B (ca. 1375−1200 BC) — the number system of classical Greece also used powers of ten, including, like the Roman numerals did, an intermediate base of 5. Notably, the polymath Archimedes (ca. 287–212 BC) invented a decimal positional system in his Sand Reckoner which was based on 10 and later led the German mathematician Carl Friedrich Gauss to lament what heights science would have already reached in his days if Archimedes had fully realized the potential of his ingenious discovery. The Hittites hieroglyphs (since 15th century BC), just like the Egyptian and early numerals in Greece, was strictly decimal.

The Egyptian hieratic numerals, the Greek alphabet numerals, the Hebrew alphabet numerals, the Roman numerals, the Chinese numerals and early Indian Brahmi numerals are all non-positional decimal systems, and required large numbers of symbols. For instance, Egyptian numerals used different symbols for 10, 20, to 90, 100, 200, to 900, 1000, 2000, 3000, 4000, to 10,000. The world's earliest positional decimal system was the Chinese rod calculus[15].



Upper row vertical form
Lower row horizontal form

Some non-mathematical ancient texts like the Vedas dating back to 1900–1700 BCE make use of decimals and mathematical decimal fractions. Some of the symbols for Brahmi numerals used by the local people[18].



Pingala, (before 2.00 B.C.) [4] in his Chanda-Sastra, used the zero symbol. He solved the problem of finding the total number of arrangements of two things in “n” places, with repetitions being allowed. The two syllables were considered “long” and “short”, denoted by I and g respectively.

To find the number of arrangements of long and short syllables in a metre containing “6” syllables, he devises the rule in short aphorisms:

	A		B
Place the number	6	Separately place	2
Halve it, result	3	Separately place	0
3 cannot be halved, therefore, subtract 1	2	Separately place	2
Halve it, result	1	Separately place	0
1 cannot be halved, therefore, subtract 1,	0	Separately place	0

Pingala knew ‘0’which means blank and, in that period, ‘0’is the symbol for Surya (Sun) too.

In the Yajttrveda Samhita, the following list of numeral 'denominations is given: Eka (I), dafa (10), sata (100), sahasra (1000), qJttta (10,000), nryuta (100,000), prqyuta (1,000,000), arbuda (10,000,000), '!Jarbuda (100,000,000), saJJ1udra (1,000,000,000), madhya (10,000,000,000), anta (100,000,000,000), parardha (1,000,000,000,000).[4]

Āryabhaṭa numeration is a system of numerals in 6th Century based on Sanskrit phonemes invented by as the name suggests, Aryabhata. In the first chapter, he assigned a numerical value to each syllable to construct every consonant and vowel possible in Sanskrit phonology, from ka = 1 up to hau = 10^{18}.

He divided the number with its place value like the Varga (Group/Class) letters ka to ma are to be placed in the varga (square) places (1st, 100th, 10000th, etc.) and Avarga letters like ya, ra, la ... have to be placed in Avarga places (10th, 1000th, 100000th, etc.).

The values of Āryabhaṭa numbers, the short vowels अ, इ, उ, ऋ, ऌ, ए, and ओ are invariably used. However, the system did not differentiate between long and short vowels. This table only cites the full slate of क-derived (1 x 10^x) values, but these are valid throughout the list of numeric syllables. [14]

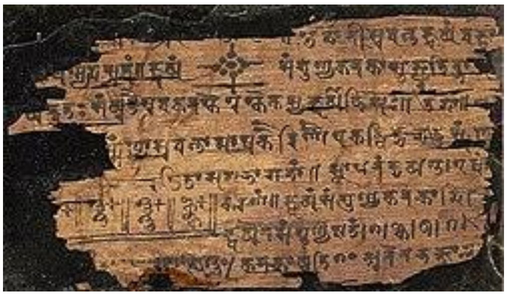
The 33 × 9 = 297 Sanskrit alphabetic numerical syllables

Nine vowels or syllabics				-a	-i	-u	-ṛ	-ḷ	-e	-ai	-o	-au	
				अ	इ	उ	ऋ	ऌ	ए	ऐ	ओ	औ	
		×		10^0	10^2	10^4	10^6	10^8	10^{10}	10^{12}	10^{14}	10^{16}	

Five velar plosives												
k **-**	क	1		क or का *ka*	कि or की *ki*	कु or कू *ku*	कृ or कृ *kṛ*	कॢ or कॢ *kḷ*	के or के *ke*	कै *kai*	को or को *ko*	कौ *kau*
kh **-**	ख	2		ख *kha*	खि *khi*	खु *khu*	खृ *khṛ*	खॢ *khḷ*	खे *khe*	खै *khai*	खो *kho*	खौ *khau*
g **-**	ग	3		ग *ga*	गि *gi*	गु *gu*	गृ *gṛ*	गॢ *gḷ*	गे *ge*	गै *gai*	गो *go*	गौ *gau*
gh **-**	घ	4		घ *gha*	घि *ghi*	घु *ghu*	घृ *ghṛ*	घॢ *ghḷ*	घे *ghe*	घै *ghai*	घो *gho*	घौ *ghau*
ṅ **-**	ङ	5		ङ *ṅa*	ङि *ṅi*	ङु *ṅu*	ङृ *ṅṛ*	ङॢ *ṅḷ*	ङे *ṅe*	ङै *ṅai*	ङो *ṅo*	ङौ *ṅau*
Five palatal plosives												
c **-**	च	6		च *ca*	चि *ci*	चु *cu*	चृ *cṛ*	चॢ *cḷ*	चे *ce*	चै *cai*	चो *co*	चौ *cau*
ch **-**	छ	7		छ *cha*	छि *chi*	छु *chu*	छृ *chṛ*	छॢ *chḷ*	छे *che*	छै *chai*	छो *cho*	छौ *chau*
j **-**	ज	8		ज *ja*	जि *ji*	जु *ju*	जृ *jṛ*	जॢ *jḷ*	जे *je*	जै *jai*	जो *jo*	जौ *jau*
jh **-**	झ	9		झ *jha*	झि *jhi*	झु *jhu*	झृ *jhṛ*	झॢ *jhḷ*	झे *jhe*	झै *jhai*	झो *jho*	झौ *jhau*
ñ **-**	ञ	10		ञ *ña*	ञि *ñi*	ञु *ñu*	ञृ *ñṛ*	ञॢ *ñḷ*	ञे *ñe*	ञै *ñai*	ञो *ño*	ञौ *ñau*
Five retroflexplosives												
ṭ **-**	ट	11		ट *ṭa*	टि *ṭi*	टु *ṭu*	टृ *ṭṛ*	टॢ *ṭḷ*	टे *ṭe*	टै *ṭai*	टो *ṭo*	टौ *ṭau*
ṭh **-**	ठ	12		ठ *ṭha*	ठि *ṭhi*	ठु *ṭhu*	ठृ *ṭhṛ*	ठॢ *ṭhḷ*	ठे *ṭhe*	ठै *ṭhai*	ठो *ṭho*	ठौ *ṭhau*
ḍ **-**	ड	13		ड *ḍa*	डि *ḍi*	डु *ḍu*	डृ *ḍṛ*	डॢ *ḍḷ*	डे *ḍe*	डै *ḍai*	डो *ḍo*	डौ *ḍau*
ḍh **-**	ढ	14		ढ *ḍha*	ढि *ḍhi*	ढु *ḍhu*	ढृ *ḍhṛ*	ढॢ *ḍhḷ*	ढे *ḍhe*	ढै *ḍhai*	ढो *ḍho*	ढौ *ḍhau*
ṇ **-**	ण	15		ण *ṇa*	णि *ṇi*	णु *ṇu*	णृ *ṇṛ*	णॢ *ṇḷ*	णे *ṇe*	णै *ṇai*	णो *ṇo*	णौ *ṇau*
Five dental plosives												
t **-**	त	16		त *ta*	ति *ti*	तु *tu*	तृ *tṛ*	तॢ *tḷ*	ते *te*	तै *tai*	तो *to*	तौ *tau*
th **-**	थ	17		थ *tha*	थि *thi*	थु *thu*	थृ *thṛ*	थॢ *thḷ*	थे *the*	थै *thai*	थो *tho*	थौ *thau*

d -	द	18		द *da*	दि *di*	दु *du*	दृ *dṛ*	दॢ *dḷ*	दे *de*	दै *dai*	दो *do*	दौ *dau*
dh -	ध	19		ध *dha*	धि *dhi*	धु *dhu*	धृ *dhṛ*	धॢ *dhḷ*	धे *dhe*	धै *dhai*	धो *dho*	धौ *dhau*
n -	न	20		न *na*	नि *ni*	नु *nu*	नृ *nṛ*	नॢ *nḷ*	ने *ne*	नै *nai*	नो *no*	नौ *nau*
Five labial plosives												
p -	प	21		प *pa*	पि *pi*	पु *pu*	पृ *pṛ*	पॢ *pḷ*	पे *pe*	पै *pai*	पो *po*	पौ *pau*
ph -	फ	22		फ *pha*	फि *phi*	फु *phu*	फृ *phṛ*	फॢ *phḷ*	फे *phe*	फै *phai*	फो *pho*	फौ *phau*
b -	ब	23		ब *ba*	बि *bi*	बु *bu*	बृ *bṛ*	बॢ *bḷ*	बे *be*	बै *bai*	बो *bo*	बौ *bau*
bh -	भ	24		भ *bha*	भि *bhi*	भु *bhu*	भृ *bhṛ*	भॢ *bhḷ*	भे *bhe*	भै *bhai*	भो *bho*	भौ *bhau*
m -	म	25		म *ma*	मि *mi*	मु *mu*	मृ *mṛ*	मॢ *mḷ*	मे *me*	मै *mai*	मो *mo*	मौ *mau*
Four approximantsor trill												
y -	य	30		य *ya*	यि *yi*	यु *yu*	यृ *yṛ*	यॢ *yḷ*	ये *ye*	यै *yai*	यो *yo*	यौ *yau*
r -	र	40		र *ra*	रि *ri*	रु *ru*	रृ *rṛ*	रॢ *rḷ*	रे *re*	रै *rai*	रो *ro*	रौ *rau*
l -	ल	50		ल *la*	लि *li*	लु *lu*	लृ *lṛ*	लॢ *lḷ*	ले *le*	लै *lai*	लो *lo*	लौ *lau*
v -	व	60		व *va*	वि *vi*	वु *vu*	वृ *vṛ*	वॢ *vḷ*	वे *ve*	वै *vai*	वो *vo*	वौ *vau*
Three coronalfricatives												
ś -	श	70		श *śa*	शि *śi*	शु *śu*	शृ *śṛ*	शॢ *śḷ*	शे *śe*	शै *śai*	शो *śo*	शौ *śau*
ṣ -	ष	80		ष *ṣa*	षि *ṣi*	षु *ṣu*	षृ *ṣṛ*	षॢ *ṣḷ*	षे *ṣe*	षै *ṣai*	षो *ṣo*	षौ *ṣau*
s -	स	90		स *sa*	सि *si*	सु *su*	सृ *sṛ*	सॢ *sḷ*	से *se*	सै *sai*	सो *so*	सौ *sau*
One glottal fricative												
h -	ह	100		ह *ha*	हि *hi*	हु *hu*	हृ *hṛ*	हॢ *hḷ*	हे *he*	है *hai*	हो *ho*	हौ *hau*

A dot was used in the place of zero in the Bakhshali Manuscript.[16] The dot symbol came to be called as the Shunya-bindu (literally, the dot of the empty place).

“The Hindu–Arabic numeral system[19] (also called the Arabic numeral system or Hindu numeral system) is a positional decimal numeral system, and is the most common system for the symbolic representation of numbers in the world. It was invented between the 1st and 4th centuries by Indian mathematicians. The system was adopted in Arabic mathematics by the 9th century. Influential were the books of Muḥammad ibn Mūsā al-Khwārizmī (On the Calculation with Hindu Numerals, c. 825) and Al-Kindi (On the Use of the Hindu Numerals, c. 830). The system later spread to medieval Europe by the High Middle Ages.”

0123456789

٠١٢٣٤٥٦٧٨٩

I II III IV V VI VII VIII IX X

०१२३४५६७८९

൦൧൨൩൪൫൬൭൮൯

౦౧౨౩౪౫౬౭౮౯

〇一二三四五六七八九

The above readings show that Aryabhata knew ‘0’, since Pingala could work on Chand Sastra.

The decimal numeration system is the common numeration system which uses ten numerals from 0 to 9. It is so far the most common and familiar numeration system which was probably developed from the human’s anatomy of ten digits in both hands. Measurement systems based on the decimal numeration are more manageable than other system because the conversion of units is by multiples of 10^n which means that it is a matter of moving the decimal point.

Hexadecimal Number System [20]

Hexadecimal Number System is the base 16 and number system uses digits 0 to 9 and A to F characters represent remaining values. This number system is used in computers.

Uses of Hexadecimal

Programmers use hexadecimal number system to simplify and ease with binary number system.

As, one hexadecimal digit = four binary digits. Thus, the hexadecimal system is used to shorten binary number system in computers.

Hexadecimals are used in the following:

To define locations in memory. Hexadecimals can characterise every byte as two hexadecimal digits when compared to eight digits while using binary.

To define colours on web pages. Each primary colour – red, green and blue is characterised by two hexadecimal digits. The format being used is #RRGGBB. RR stands for red, GG stands for green and BB stands for blue.

To represent Media Access Control (MAC) addresses. MAC addresses consist of 12-digit hexadecimal numbers. The format being used is either MM:MM:MM:SS:SS:SS or MMMM-MMSS-SSSS. The first 6 digits of the MAC address represent the ID of the adapter manufacturer while the last 6 digits represent the serial number of the adapter.

To display error messages. Hexadecimals are used to define the memory location of the error. This is useful for programmers in finding and fixing errors.

Advantages of the Hexadecimal System

Here are some advantages of using the hexadecimal system:

"It is very concise and short. By using a base of 16 means that the number of digits used to signify a given number is usually less than in binary or decimal. It allows us to store more information using less space.

It is fast and simple to convert between hexadecimal numbers and binary. Hexadecimal can be used to write large binary numbers in just a few digits.

It makes life easier as it allows grouping of binary numbers which makes it easier to read, write and understand. It is more user-friendly, as humans are used to group together numbers and things for easier understanding. Also, writing in less digits lowers the possibility of error occurring."

Bibliography

1. A.B. Taylor, " Report on Weights and Measures", *Pharmaceutical Association, 8th Annual Session*, Boston, Sept. 15, 1859. See pages and 48 and 53.
2. Alfred B. Taylor, "Octonary numeration and its application to a system of weights and measures", *Proc. Amer. Phil. Soc.* Vol XXIV, Philadelphia, 1887; pages 296-366. See pages 327 and 330.
3. Bender, Andrea; Beller, Sieghard ,. "Mangarevan invention of binary steps for easier calculation". *Proceedings of the National Academy of Sciences.* 111: 1322–1327.
4. Datta and Singh(1935), *History of Hindu Mathematics*, Asia Publishing House ,Bombay,New York,New Delhi,London.
5. Fleet, J. F. (1911). "Aryabhata's System of Expressing Numbers". *The Journal of the Royal Asiatic Society of Great Britain and Ireland. Royal Asiatic Society of Great Britain and Ireland*. 43: 109–126.
6. H.R. Phalen, "Hugh Jones and Octave Computation," *The American Mathematical Monthly* 56 (August–September 1949): 461-65.
7. Leibniz G., "Explication de l'Arithmétique Binaire", *Die Mathematische Schriften, ed. C. Gerhardt*, Berlin 1879, vol.7, p.223; Engl. transl.
8. Marcia Ascher. "Ethnomathematics: A Multicultural View of Mathematical Ideas". *The College Mathematics Journal*. JSTOR 2686959.
9. Ryan, James A. , "Leibniz' Binary System and Shao Yong's "Yijing"". *Philosophy East and West*. University of Hawaii Press. 46 (1): 59–90..

Web Bibliography

10. Agarwal,Sarita(2009), "Number Conversion", *www.wikieducator.org/User:Sarita_agarwal.*
11. *http;//en.wikipedia.org/wiki/Binary_Number*
12. *http;//en.wikipedia.org/wiki/Binary_Code*
13. *http;//en.wikipedia.org/wiki/Octal*
14. *http;//en.wikipedia.org/wiki/Aryabhata_Numeration*
15. *http;//en.wikipedia.org/wiki/Decimal*
16. *http;//en.wikipedia.org/wiki/Bakhshali_manusript*
17. *http;//www.storyofmathematics.com/Indian_brahamagupta_html*
18. *http;//en.wikipedia.org/wiki/Brahmi_numerals*
19. *http;//en.wikipedia.org/wiki/Hindu-Arabic_Number_System*

20. *http;//en.wikipedia.org/wiki/Hexadecimal*