Two-Sample t-Test: Independent Samples

Chapter 14

Overview of Tests

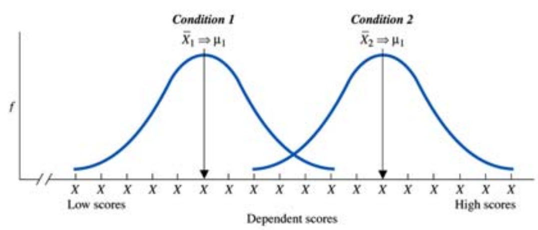
- z-Test
- One Sample t-Test

Compare one sample to a population

- Related Samples t-Test
- Independent Samples t-Test

Compare two samples

Difference between Means in a Two-sample Experiment



Independent Samples t-Test

- Independent when we randomly select participants for samples
- Assumptions
  - dependent variable has interval or ratio scores
  - populations of raw scores form at least roughly normal distributions
  - Homogeneity of variance
    - variance (σ^2_X) of each population being represented is equal
  - n's don't have to be equal, as long as they're not radically different (e.g. n_1 more than twice as much as n_2)

Calculating t_{obt} for Repeated Measures

$$t_{obt} = \frac{\overline{D}}{s_{\overline{D}}}$$

Calculating t_{obt} for Independent samples

$$t_{obt} = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}_1 - \overline{X}_2}}$$

Photos and False Memories

Research Question:

- Can old photos create or prevent false memories?

(Lindsay, Hagen, Read, Wade, & Garry, 2004)

- 45 undergraduates asked to remember a childhood event that, unbeknownst to them, hadn't actually occurred.
  - one group ($n = 23$) given photograph to help cue their memory
  - second group ($n = 22$) not given photograph
- Participants rated the extent to which their memory experience resembled "reliving the event" on a scale from 1 to 7

  (1=not at all, 7 = as if it were happening right now)

Photos and False Memories

Photo group $\overline{X} = 3.22$

Control group $\overline{X} = 2.00$

A 6 Step Program for Hypothesis Testing

1. State the research question
2. Choose a statistical test
3. Select alpha which determines the critical value for the region of rejection (e.g., $t_{.05}$)
4. State your statistical hypotheses (as equations)
5. Collect data, and calculate t_{obt}
6. Interpret results in terms of hypothesis
   Report results
   Explain in plain language

A 6 Step Program for Hypothesis Testing

1. State your research question
   - Can old photos influence false memories scores?
2. Choose a statistical test
   - comparing means from two groups
   - participants randomly assigned to different groups
   - independent sample *t*-Test

Photos and False Memories

3. Select alpha which determines the critical value ($t_{.05}$) for the region of rejection
   - $\alpha = .05$
   - For the independent two-sample *t*-Test
     - $df = (n_1 - 1) + (n_2 - 1)$
     - in other words, $df = N - 2$
     - in this case, $df = 45-2 = 43$
   - $t_{.05} = +/- 2.021$ (from *t*-Tables)

Photos and False Memories

4. State your statistical hypotheses (as equations)

   H_0: $\mu_1 = \mu_2$

   H_1: $\mu_1 \neq \mu_2$

   When H_0 is true, both samples represent the same population

5. Collect data and calculate test statistic (t_{obt})

	Photo	**Control**
n	23	22
$\overline{X}$	3.22	2.00
s_x	1.575	1.341

$$t_{obt} = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}_1 - \overline{X}_2}}$$

Is the difference in means due to the photos or simply be due to chance variation?

Properties of the Difference between Means

- Sampling distribution of difference between means
  - distribution of all possible differences between two means
- Standard error of the difference between means
  - standard deviation of this sampling distribution

$$s_{\bar{X}_1 - \bar{X}_2}$$

Collect data and calculate test statistic (t_{obt})

One sample *t*-Test

$$t_{obt} = \frac{\bar{X} - \mu}{s_{\bar{X}}}$$

standard error

Independent samples *t*-Test

$$t_{obt} = \frac{\bar{X}_1 - \bar{X}_2}{s_{\bar{X}_1 - \bar{X}_2}}$$

standard error of the difference

Collect data and calculate test statistic (t_{obt})

Estimating population variance requires two steps:

1. Compute pooled variance *(a weighted average)*

$$s^2_{pool} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

2. Compute standard error of the difference

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{(s^2_{pool})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Pooled Variance

$$s^2_{pool} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

- The average of both sample variances, once adjusted for their degrees of freedom (*df*)
- You have made a mistake if pooled variance
  - does not come out between the two estimates
  - does not come out closer to the estimate from the larger sample

Pooled Variance

	Photo	Control
n	23	22
$\bar{X}$	3.22	2.00
s_x	1.575	1.341
s_x^2	2.481	1.798

$$s^2_{pool} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

$$s^2_{pool} = \frac{(23-1)(2.481) + (22-1)(1.798)}{(23-1) + (22-1)}$$

$$s^2_{pool} = 2.147$$

Standard Error of the Difference

	Photo	Control
n	23	22
$\bar{X}$	3.22	2.00
s_x	1.575	1.341
s_x^2	2.481	1.798

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{(s^2_{pool})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{(2.147)\left(\frac{1}{23} + \frac{1}{22}\right)}$$

$$s_{\bar{X}_1 - \bar{X}_2} = .437$$

Calculate test statistic (t_{obt})

	Photo	Control
n	23	22
$\overline{X}$	3.22	2.00
s_x	1.575	1.341
s_x^2	2.481	1.798

$$t_{obt} = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}_1 - \overline{X}_2}}$$

$$t_{obt} = \frac{3.22 - 2.00}{.437}$$

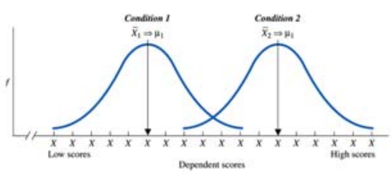
$$t_{obt} = 2.79$$

Photos and False Memories

- Interpret results in terms of hypothesis
  - $2.79 > 2.02$; Reject H_0
- Report results
  - $t(43) = 2.79$, $p < .05$
- Explain in plain language
  - The photo group (M = 3.2, SD = 1.56) produced significantly higher ratings of "reliving" the false event than the control group (M = 2.0, SD = 1.34).
  - When combined with other suggestive techniques, old photos can contribute to false memories.

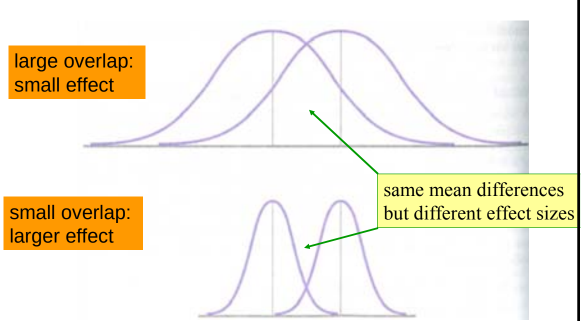
Effect Size

- Significant effects may indicate genuine, but trivial differences between groups
- Effect size indicates the size of a difference between groups
  - the less overlap the larger the difference



- Effect size is influenced by
  - separation between means

- Effect size also influenced by
  - variation in each group



Interpreting Cohen's *d*

- These are rough guidelines:

Effect Size	*d*	Overlap
small	.2	85%
medium	.5	67%
large	.8	53%

Effect Size: Cohen's *d*

- Cohen's *d*
  - expresses the difference between means in standard deviation units

$$\hat{d} = \frac{mean1 - mean2}{SD}$$

for independent samples: use weighted average standard deviation (square root of pooled variance)

$$\hat{d} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{s^2_{\text{pool}}}}$$

Note: do not calculate effect size for **nonsignificant** results

Effect Size

- Cohen's *d*
  - alternate equation
  - use this equation when calculating effect size from PASW output

$$\hat{d} = \frac{2t}{\sqrt{df}}$$

t_{obt}

Note: do not calculate effect size for **nonsignificant** results

Photos and False Memories

- What is the effect size in the false memories study?

$\overline{X}_1 = 3.22 \quad \overline{X}_2 = 2.00 \quad s^2_{pool} = 2.147$

$$\hat{d} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{s^2_{\text{pool}}}} \qquad \hat{d} = \frac{3.22 - 2.00}{\sqrt{2.147}} = .83$$

Interpreting PASW printouts

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means					95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
scores	Equal variances assumed	176.056	.000	-1.769	10	.107	-12.66667	7.16163	-28.62376	3.29043
	Equal variances not assumed			-1.769	5.880	.128	-12.66667	7.16163	-30.27769	4.94435

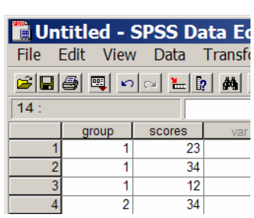
t	df	Sig. (2-tailed)
-1.769	10	.107

Report: $t(10) = -1.77, p = .107$

Note: when using PASW, report the actual *p* - values

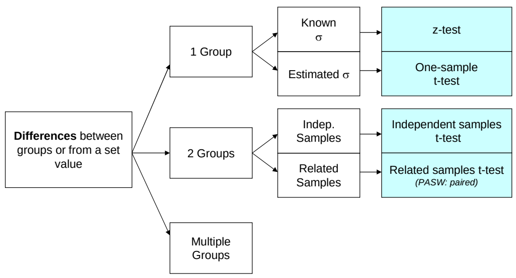
Independent Samples *t*-Test and PASW

- Create two variables
  - one contains levels of your independent variable (here called "**group**")
  - the second contains scores of your dependent variable (here called "**scores**')



Independent Samples *t*-Test and PASW

- Select from Menu:
  Analyze -> Compare Means -> Independent Samples T Test
- Select your dependent variable (e.g., scores) as *Test Variable* and independent variable (e.g., group) as *Grouping Variable*.
- Select "Define Groups"
- Enter "1" for Group 1 and "2" for Group 2
  - Note: you would enter different labels if you had not named your groups 1 and 2
- Click Continue; Click OK



- People that take Lipitor have reduced levels of cholesterol.

- If researchers find a difference between groups, why do the two groups differ?
  1. random differences between individuals in the groups.
     - e.g., more healthy people in one group
  2. differences between levels of the independent variable
     - e.g., Lipitor is effective in reducing cholesterol level